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Optimizing an Advertising Campaign

Math 1010 Intermediate Algebra Project

Background Information:
Linear Programming is a technique used for optimization of a real-world situation. Examples of optimization include maximizing the number of items that can be manufactured or minimizing the cost of production. The equation that represents the quantity to be optimized is called the objective function, since the objective of the process is to optimize the value. In this project the objective is to maximize the number of people who will be reached by an advertising campaign.

The objective is subject to limitations or constraints that are represented by inequalities. Limitations on the number of items that can be produced, the number of hours that workers are available, and the amount of land a farmer has for crops are examples of constraints that can be represented using inequalities. Broadcasting an infinite number of advertisements is not a realistic goal. In this project one of the constraints will be based on an advertising budget.

Graphing the system of inequalities based on the constraints provides a visual representation of the possible solutions to the problem. If the graph is a closed region, it can be shown that the values that optimize the objective function will occur at one of the "corners" of the region.

The Problem:

In this project you will solve the following situation:

A local business plans on advertising their new product by purchasing advertisements on the radio and on TV. The business plans to purchase at least 60 total ads and they want to have at least twice as many radio ads as TV ads. Radio ads cost $50 each and TV ads cost $200 each. The advertising budget is $10,800. It is estimated that each radio ad will be heard by 2000 listeners and each TV ad will be seen by 1500 people. How many of each type of ad should be purchased to maximize the number of people who will be reached by the advertisements?

Modeling the Problem:

Let X be the number of radio ads that are purchased and Y be the number of TV ads.

1. Write down a linear inequality for the total number of desired ads.

$x + y \geq 60$

2. Write down a linear inequality for the cost of the ads.

$50x + 200y \leq 10{,}800$

3. Recall that the business wants at least twice as many radio ads (X) as TV ads (Y). So, the number of radio ads should be at least (or more) than 2 times the number of TV ads. Write down a linear inequality that expresses this fact. Be careful! It is easy to write this one backwards.

   $2x + y \geq 60$

4. There are two more constraints that must be met. These relate to the fact that there cannot be s negative numbers of advertisements. Write the two inequalities that model these constraints:

   $x \geq 0$ & $y \geq 0$

5. Next, write down the function for the number of people that will be exposed to the advertisements. This is the Objective Function for the problem.

   $P = 2000x + 1500y$

You now have five linear inequalities and an objective function. These together describe the situation. This combined set of inequalities and objective function make up what is known mathematically as a **linear programming** problem. Write all of the inequalities and the objective function together below. This is typically written as a list of constraints, with the objective function last.

$x + y \geq 60$

$50x + 200y \leq 10{,}800$

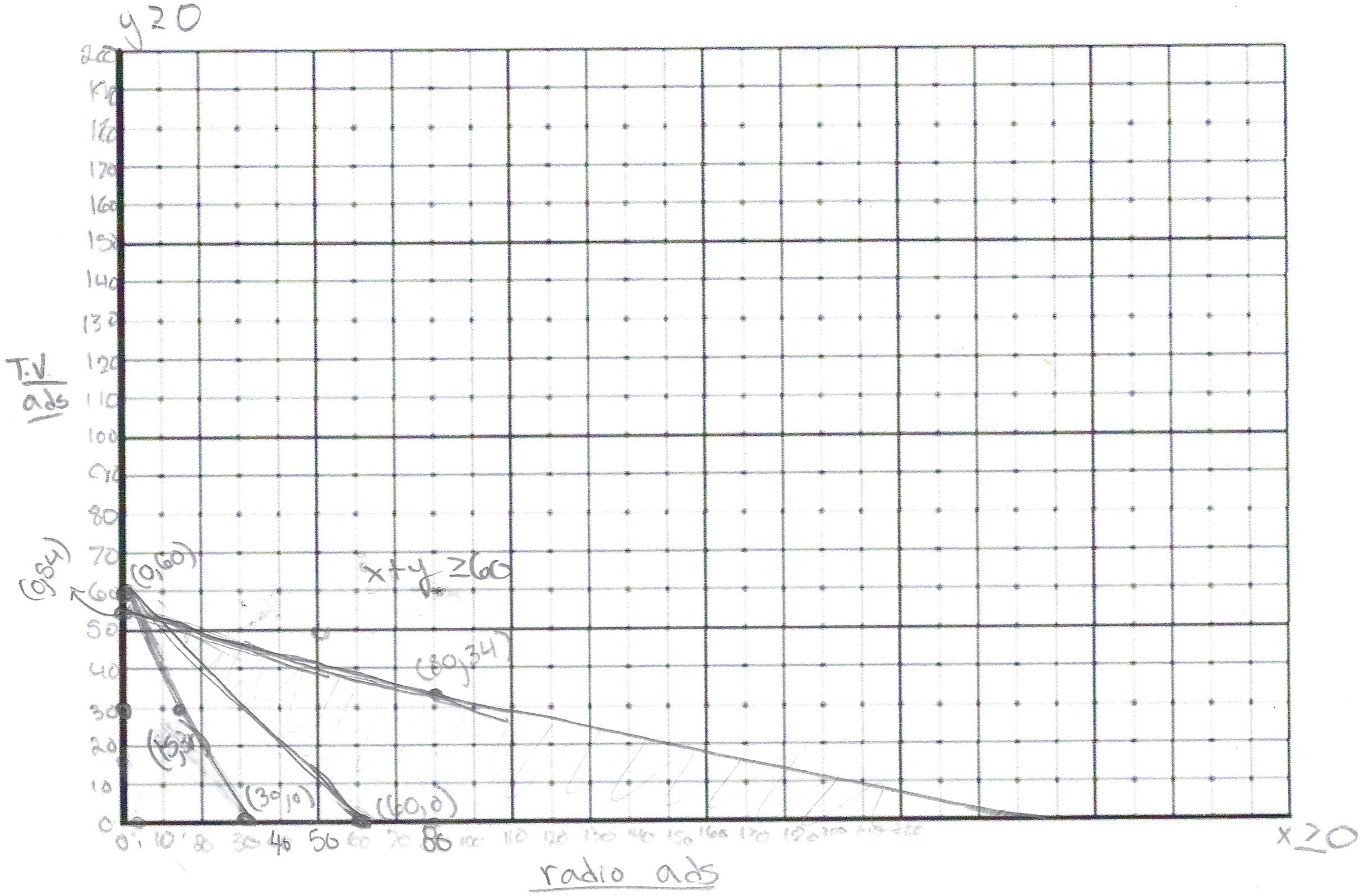
$2x + y \geq 60$

$x \geq 0$

$y \geq 0$

$P = 200x + 1500y$

6. To solve this problem, you will need to graph the **intersection** of all five inequalities on one common XY plane. Do this on the grid below (or for extra credit use www.desmos.com. Make sure you scale your graph appropriately. You'll want to export your graph as an "image".). Have the bottom left be the origin, with the horizontal axis representing X and the vertical axis representing Y. Label the axes with what they represent and label your lines as you graph them.



1. $x + y \geq 60$

x	y
0	60
60	0

2. $\frac{50x + 200y}{50} \leq \frac{10{,}800}{50}$

$x + 4y \leq 216$

$-4y \quad -4y$

$x \leq 216 - 4y$

$0 \geq 216 - 4y$

$-216 \quad -216$

$-216 \leq -4y$

$\frac{-216}{-4} \quad \frac{-4y}{-4}$

$y \geq 54$

x	y
0	54
80	34

3. $2x + y \geq 60$

$-2x \quad -2x$

$y \geq 60 - 2x$

$-60 \quad -60$

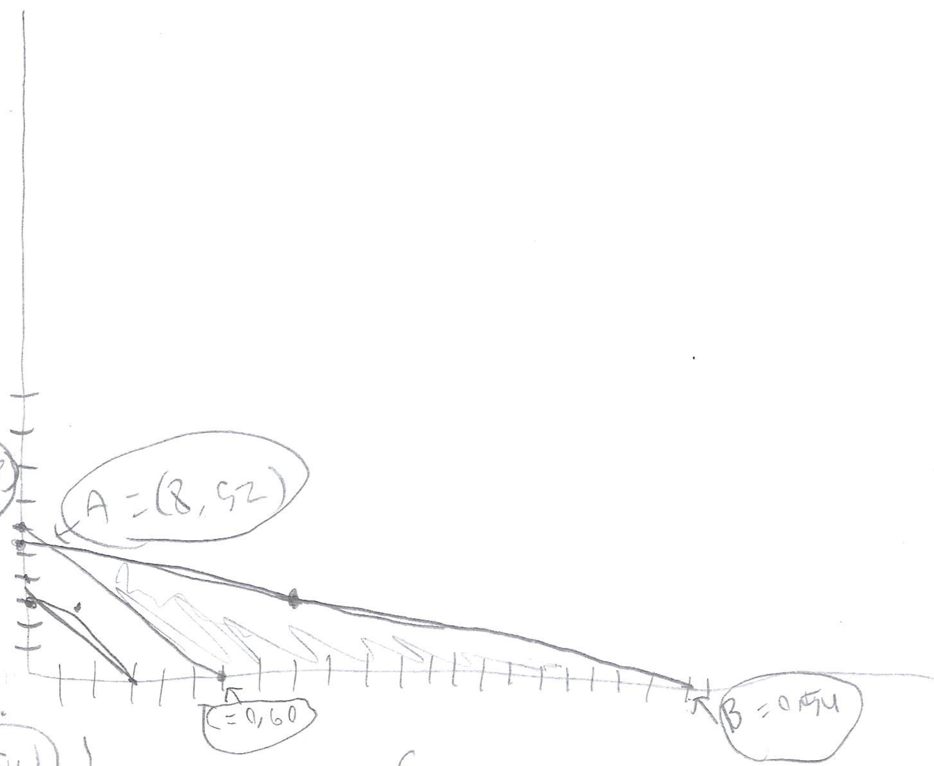
x	y
15	30
30	0

7. The shaded region in the above graph is called the feasible region. Any (x, y) point in the region corresponds to a possible number of radio and TV ads that will meet all the requirements of the problem. However, the values that will maximize the number of people exposed to the ads will occur at one of the vertices or corners of the region. Your region should have three corners. Find the coordinates of these corners by solving the appropriate system of linear equations. Be sure to *show your work* and *label* the (x, y) coordinates of the corners in your graph.

$x + y = 60$
$-(x + 4y = 216)$
$3y = 156$
$y = 52$

$x + 52 = 60$
$-52 \quad -52$
$x = 8$

A. (8, 52)



$x = 0$
$x + 4y = 216$
$4y = 216$
$y = 54$

B. (0, 54)

$x + y = 60$
$x = 0$
$y = 60$

C. (0, 60)

8. To find which number of radio and TV ads will maximize the number of people who are exposed to the business advertisements, evaluate the objective function P for each of the vertices you found. Show your work.

$P = 2000x + 1500y$

A. $2000(8) + 1500(52) = 94000$

B. $2000(0) + 1500(54) = 81000$

C. $2000(0) + 1500(60) = 90000$

9. Write a sentence describing how many of each type of advertisement should be purchased and what is the maximum number of people who will be exposed to the ad.

THEY SHOULD PURCHASE 8 T.V. ADS & 52 RADIO ADS
TO BE ABLE TO REACH THE MOST PEOPLE, A 94000

10. **Reflective Writing**. This needs to be a separate page that is typed, proof-read for typos, spelling, and grammar. Use 12-point font and double space. Add a title (e.g. Reflective Writing for Optimization Lab). Provide a brief introduction explaining the lab in your own words. Also in the introduction, tell the audience which mathematical techniques you used in the lab (e.g. plotting graphs). Then, please respond to each of the questions below. Your writing should be in an essay form (written in paragraphs).

    Do you think this project shows how math can be applied to the real world? If "yes", please elaborate-why are the results important or beneficial? If "no", how could the lab change to make it more applicable to the "real world"? Can you give an example of someone other than a business owner/manager that might be interested in this type of analysis? Can you think of or find an example of in your text (Hint: see the exercise set for section 4.5 in your text.) another application where linear programming could be used to optimize a situation? Be specific. If you were the business owner in this lab and the manager brought you the results of the lab, why would it be important for the manager to be clear about the process and be able justify her conclusion? Did this assignment change your opinion of the usefulness of math? Write one paragraph stating what ideas changed and why. If this project did not change the way you think, write how this project gave further evidence to support your existing opinion about applying math. Be specific.

11. ePortfolio. Post a copy of this lab, including the Reflective Writing, to your ePortfolio (if you need to, scan a copy in the Copy Center). For more information about ePortfolios, please see the syllabus.

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Professor Paxton

MATH 1010

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Reflective Writing on Optimization Project

This lab was about figuring out how many T.V. and Radio ads should be purchased to reach the maximum amount of people. I used solving system of equations, plotting points on a graph, and solving inequalities.

This project most definitely shows how math can be used in the real world. The results are important because it tells the company exactly how many ads they need to purchase for maximum amount of people reached, spending the most minimal amount of money. I think someone who owns a small business could use this analysis to grow his business and become more well known. I cannot think of a different application, or find one in the book. He should be clear about the results so I could trust that he did everything correctly and efficiently with the right numbers.

This project did change how I think of math in the real world. It makes more sense of reasons that I can use math to figure out things that I will need to do in life. I think I will be able to figure out more situations and help others as well.